

Article I



PRIME
NUMBERS

$$H(a_+ \psi) = (E - \hbar\omega)(a_+ \psi) \quad J(x) = \sum_{n=1}^{\infty} c_n \psi_n(x) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right) \quad \text{Nuclear radius} = A^{1/3} \cdot 1.2 \text{ fm}$$

$$\frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = -m^2 \quad \text{solenoid: } L = N\Phi/I = \mu_0 AN^2/\ell. \quad \tau_{1/2} = \ln(2)\tau, \quad N = N_0 \exp(-t/\tau)$$

$$H = \hbar\omega \left(a_+ a_- + \frac{1}{2} \right) \quad PE = -G \frac{Mm}{r}, \quad \Delta PE = mgh (\text{small } h), \quad F = G \frac{Mm}{r^2} = mg$$

$$p_x \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x}, \quad p_y \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial y}, \quad p_z \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial z} \quad B\ell = \mu_0 I \text{ for single wire } B = \frac{\mu_0 I}{2\pi r} \quad c_n = \int \psi_n(x)^* f(x) dx$$

$$U_{\text{capacitor}} = Q^2/(2C) = CV^2/2 =$$

Quantum Mechanics:

$$L = I\omega = mvr \sin \theta, \quad (\theta = \text{angle between } v \text{ and } r)$$

$$a_+ \equiv \frac{1}{\sqrt{2\hbar m\omega}} (-ip + m\omega x) \quad U = \epsilon_0 E^2/2 + B^2/(2\mu_0) = \text{energy/volume}$$

$$n_a \sin \theta_a = n_b \sin \theta_b, \quad \sin \theta_{\text{crit}} = \frac{n_b}{n_a} \quad \Delta L/L = \alpha \Delta T, \quad \Delta V/V = 3\alpha \Delta T$$

$$\Theta(\theta) = AP_l^m (\cos \theta) \quad \lambda_{\text{matter}} = \lambda_{\text{vac}}/n, \quad f_{\text{matter}} = f_{\text{vac}}, \quad c_{\text{matter}} = c_{\text{vac}}/n$$

$$\tau = rF \sin \theta, \quad I\alpha = \tau, \quad I_{\text{point}} = mR^2 \quad v = \omega r = \frac{2\pi r}{T}, \quad \omega = 2\pi f = \frac{2\pi}{T}, \quad f = 1/T$$

$$L = \hbar \sqrt{\ell(\ell + 1)}, \quad L_z = m_\ell \hbar, \quad m_\ell = -\ell, \dots, \ell \quad \Psi_n(\mathbf{r}, t) = \psi_n(\mathbf{r}) e^{-iE_n t/\hbar}$$

$$\int \psi_m(x)^* \psi_n(x) dx = \delta_{mn} \quad F = qvB \sin \theta, \quad F = ILB \sin \theta$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) \quad \delta_{mn} = \begin{cases} 0, & \text{if } m \neq n \\ 1, & \text{if } m = n \end{cases}$$

$$\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C} \quad \rho = m (\text{unit: kg/m}^3) V \quad \hbar\omega \left(a_+ a_\pm \pm \frac{1}{2} \right) \psi = E\psi$$

Black body: $\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m}\cdot\text{K}$ $h = 6.626 \times 10^{-34}$

Article I: On the number of prime numbers between n^2 and $(n+1)^2$

The proposed article will be a scientific announcement concerning the discovery of a surprising functional dependence that determines the pattern of distribution of prime numbers in short intervals:

Below we propose a model presenting the cascade mechanism of prime numbers generation, formal notation of which is illustrated by the following theorem:

Suppose k is the amount of prime numbers between n and $2n+1$, while c is the amount of prime numbers between n^2 and $(n+1)^2$

Theorem: Between n^2 and $(n+1)^2$ exist c prime numbers, c converges asymptotically to the value $\approx k$

For readers who do not deal professionally with mathematical reasoning, the sense of this theorem is clearer when it is illustrated using specific numerical examples:

10 – 21 (It is the interval containing 10 numbers, including 4 prime numbers: 11,13,17 and 19, that is k prime numbers)

↓ **$10^2=100, 11^2=121 \rightarrow 100-121$** (the interval contains 20 numbers, including 5 prime numbers: 101, 103, 107, 109 and 113, that is c prime numbers)

100 – 201 ($k = 21$ prime numbers)

↓↓ **$100^2=10,000, 101^2 = 10,201 \rightarrow 10,000 - 10,201$**
($c = 23$ prime numbers)

1000 – 2001 ($k = 135$ prime numbers)

↓↓↓ **$1000^2=1,000,000, 1001^2 = 1,002,001 \rightarrow 1,000,000 - 1,002,001$**
($c = 152$ prime numbers)

10,000 – 20,001 ($k = 1037$ prime numbers)

↓↓↓↓ **$10\,000^2=100,000,000, 10,001^2=100,020,001 \rightarrow 100,000,000 - 100,020,001$**
($c = 1089$ prime numbers)

100,000 – 200,001 ($k = 8391$ prime numbers)

↓↓↓↓↓ **$100,000^2=10,000,000,000, 100,001^2=10,000,200,001 \rightarrow 10,000\,000,000-10,000,200,001$**
($c = 8631$ prime numbers)

Our research encompassed over 100,000 intervals of n^2 and $(n+1)^2$. The discovery of the said functional dependence has enabled the formalisation in form of the theorem presented above. There are, of course, certain important questions concerning the discovered dependence and the formulated theorem. We shall try to refer to these questions in our following articles.

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Abstract:

Prime numbers in short intervals. The Cywiński - Książek theorem on the number of prime numbers between: n^2 and $(n+1)^2$. D. Andrica's theorem.

Considering the importance of the discovered functional dependence in the research on the frequency of prime numbers occurrence in short intervals, we have decided to include only a raw result herein (frame contains the entire article), so that it could reach a wide group of researchers as soon as possible. The relatively adequate dependence discovered by Gauss, has been titled the Prime Number Theorem. This form of presentation should facilitate and accelerate the reviewing process. Below we attach a compilation of opinions of leading mathematicians, given in interviews, e.g. for BBC, that provide the best justification for the essence of the discovered dependence:

"[...] However, the knowledge on *the distribution of prime numbers still remains a mystery for the researchers*. That fact, of course, only deepens frustration of the researchers: *we do not even know what would have to be proven in order to resolve the problem*. This frustration is reflected in the researchers' question: *"Is there a way to understand [how prime numbers are distributed], if not completely, then at least to the degree that would allow to determine the pattern that they impose on mathematics?"* It was followed by a period of waiting for a new, revolutionary idea that could indicate the path for further research.[...]"

The analogy of the proposed model and the models proposed by Watson and Crick or Mendeleev, that have set the direction of research in genetics and chemistry for decades, is now obvious. Therefore the heuristic explanation - in form of essays comprehensible for the general scientific community - describing the mechanism of generation of prime numbers and providing an effective proof, shall be presented in our following articles.